

Assuming 4-core CPU chip with total die area=3.1cm², defect rate=0.6 defect/cm², and $\alpha=4$.

Question: Find the probability that at least 3 out of 4 cores are working (so you can sell the chip as a 3-core CPU).

Solution:

We can assume 4 independent cores, each with area=0.775cm².

$$\text{DieYield}_{\text{core}} = 1 * ((1 + 0.6 * 0.775 / 4))^{-4} = 0.644$$

This problem is similar to tossing an unfair coin, where P(heads)=0.644, P(tails)=0.356. The probabilities of each core containing a defect are independent.

Say G=good core, B=bad core, and C₁C₂C₃C₄ = status of each core (G or B)

$$P(0 \text{ defect}) = P(\text{GGGG}) = 0.644^4 = \mathbf{0.172}$$

P(1 defect) = P(GGGB U GGBG U GBGG U BGGG) = P(GGGB) + ... + P(BGGG) because probabilities are independent.

$$\begin{aligned} \text{The individual scenario GGGB has } P(\text{GGGB}) &= P(\text{GGBG}) = P(\text{GBGG}) = P(\text{BGGG}) \\ &= 0.644^3 * 0.356^1 = 0.095 \end{aligned}$$

$$\text{so } P(1 \text{ defect}) = (4 \text{ choose } 1) * 0.095 = 0.380$$

$$\text{finally, the problem asks for } P(1 \text{ defect U } 0 \text{ defect}) = P(1 \text{ defect}) + P(0 \text{ defect}) = 0.172 + 0.380 = \mathbf{0.552}.$$

Question: Find the probability that at least 2 out of 4 cores are working (so you can sell the chip as a 2-core, 3-core or 4-core CPU).

$$P(0 \text{ defect}) = 0.172$$

$$P(1 \text{ defect}) = 0.380$$

$$\begin{aligned} P(2 \text{ defects}) &= (4 \text{ choose } 2) * P(\text{GGBB}), \text{ since all permutations are equally likely} \\ &= 6 * (0.644^2 * 0.356^2) = 0.315 \end{aligned}$$

$$\text{The problem asks for } P(2 \text{ defects or less}) = 0.315 + 0.380 + 0.172 = \mathbf{0.867}$$

Question: Find the probability that at least 1 out of 4 cores are working (so you can sell the chip as a 1-core, 2-core, 3-core or 4-core CPU).

$$P(0 \text{ defect}) = 0.172$$

$$P(1 \text{ defect}) = 0.380$$

$$P(2 \text{ defects}) = 0.315$$

$$\begin{aligned} P(3 \text{ defects}) &= (4 \text{ choose } 3) * P(\text{GGBB}), \text{ since all permutations are equally likely} \\ &= 4 * (0.644^1 * 0.356^3) = 0.116 \end{aligned}$$

$$\text{The problem asks for } P(3 \text{ defects or less}) = 0.315 + 0.380 + 0.172 + 0.116 = \mathbf{0.983}$$